mCRL2

Towards a practical formal specification language

Aad Mathijssen Jan Friso Groote Muck van Weerdenburg

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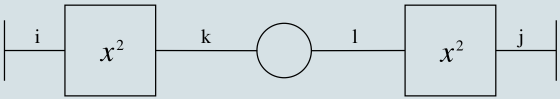
Motivation: Petri Nets

Bring stand-alone developments of specification languages together.
GenSpect: find a common base for hierarchical Petri Nets and process algebra with data.



It should be possible to translate Petri Nets to process algebra:

- places are unordered buffers
- transitions are memoryless input/output relations
- arcs define communication between places and transitions

Motivation: Petri Nets (2)

We would like to use μCRL as a target for this translation. Unfortunately, there are a number of problems:

- all actions involved in the firing of transitions occur at the same time
- hierarchical approach enforces that operators are compositional, but communication is not

Motivation: concrete data types

Problems with the use of μCRL in practise, because of the lack of *concrete data types*:

- specifications are too long
- standard notions are specified differently amongst different specifications
- lack of higher-order notions

Specifying all data types yourself distracts from doing the real work.

Motivation: linear process equations

Every guarded untimed μCRL specification can be transformed to a linear process equation (LPE), which has the following form:

$$P(\overrightarrow{d{:}D}) = \sum_{i \in I} \sum_{\overrightarrow{e_i:E_i}} a_i(\overrightarrow{f_i}(\overrightarrow{d, e_i})) \cdot P(\overrightarrow{g_i}(\overrightarrow{d, e_i})) \lhd c_i(\overrightarrow{d, e_i}) \rhd \delta$$

An LPE is a symbolic representation of a state space.
It is the core language used by the μCRL toolset.

Two things are lacking:

- time
- don't care values

mCRL2

Design a new language and toolset, using both theoretical and practical experience with μCRL. Basically, the mCRL2 language is timed μCRL with the following changes/additions:

- true concurrency (multi-actions)
- local communication
- higher-order algebraic specification
- concrete data types

The toolset will use a new LPE format, which supports multi-actions, higher-order algebraic specification, time and don't care values.

mCRL2 (2)

To find out if the language and the toolset is useful in practise, we took the following approach to design the language:

1. start with an initial design of the language and a toolset
2. iteratively:
   (a) test using real-world examples
   (b) improve formal language
   (c) improve toolset

mCRL2 process language

Process expressions have the following syntax:

$$
\begin{array}{rl}
p ::= & a(\vec{d}) \mid \delta \mid \tau \mid p + p \mid p \cdot p \mid p \parallel p \mid p \mathbin{⌊\!⌊} p \mid p|p \mid X(\vec{d}) \\
& \mid (d = d) \rightarrow p, p \mid p \cdot d \mid \sum_{\vec{x}:\vec{s}} p \\
& \mid \nabla_V(p) \mid \partial_{IH}(p) \mid \tau_{IH}(p) \mid \Gamma_C(p) \mid \rho_R(p)
\end{array}
$$

- sync operator $|$ does not communicate
- a sync of actions is called a *multi-action*, e.g. $a,\ a|b,\ b|a,\ a|b|c,\ a|b|a,\ a(t)|b(u)|a(v)$
- V and IH are sets of parameterless multi-actions/actions
- C and R are sets of renamings of parameterless multi-actions/actions to actions; the lhs's of C/R must be disjoint

mCRL2 process language (2)

Communication and restriction:

- communication operator Γ_C realises communication of multi-actions with equal parameters, e.g. where $t = u$ and $t \neq v$:
  $\Gamma_{\{a|b \to c\}}(a(t)|b(u)) = c(t),\ \Gamma_{\{a|b \to c\}}(a(t)|b(v)) = a(t)|b(v)$,
  $\Gamma_{\{a|b|c \to d\}}(a|b|c|d) = d|d,\ \Gamma_{\{a|b|c \to d, d|d \to d\}}(a|b|c|d) = d|d$
  $\sum_{d:D} \Gamma_{\{a|a \to a\}}(a(d)|a(t)) = \sum_{d:D} d = t \to a(t), a(d)|a(t)$
- visibility operator ∇_V *only* multi-actions that are in the set V, e.g.
  $\nabla_{\{a,b\}}(a \parallel b) = a \cdot b + b \cdot a,\ \nabla_{\{a|b\}}(a \parallel b) = a|b$,
  $\nabla_{\{a|b\}}(a|b|c) = \delta,\ \nabla_{\{a,b|c\}}(a \parallel b \parallel c) = a \cdot (b|c) + (b|c) \cdot a$
- blocking operator ∂_{IH} blocks all actions that occur in the set IH, e.g.
  $\partial_{\{a\}}(a + b \cdot (a|c)) = b \cdot \delta$

mCRL2 process language (3)

Process equations are formed as follows:

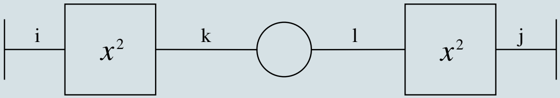
$$pe ::= X(\overrightarrow{x : s}) = p$$

Process specifications:

$$sp ::= (\textbf{act}\ (a;\ |\ a : s \times \cdots \times s;\ )^{+}\ |\ \textbf{proc}\ (pe;\ )^{+})^{*}\ \textbf{init}\ p;$$

Petri Net translation

Petri Nets can be expressed in mCRL2:



Translation to mCRL2:

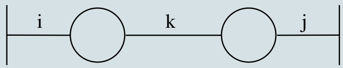
$$
\begin{aligned}
Sqr_{i,o} &= \textstyle\sum_{n:\mathbb{N}} \overline{get_i}(n) | \overline{put_o}(n^2) \cdot Sqr_{i,o} \\
P_{i,o}(b : Bag(\mathbb{N})) &= \textstyle\sum_{n:\mathbb{N}} \underline{put_i}(n) \cdot P_{i,o}(b \cup \{ n \}) + \\
&\quad \textstyle\sum_{n:\mathbb{N}} n \in b \rightarrow \underline{get_o}(n) \cdot P_{i,o}(b \setminus \{ n \}) \\
DSqr_{i,j} &= \nabla_V(\Gamma_C(Sqr_{i,k} \parallel \overline{P_{k,l}(\emptyset)} \parallel Sqr_{l,j}))
\end{aligned}
$$

where

$$
C = \{ \overline{put_k} | \underline{put_k} \rightarrow put_k, \overline{get_l} | \underline{get_l} \rightarrow get_l \}, V = \{ \overline{get_i} | put_k, get_l | \overline{put_j} \}
$$

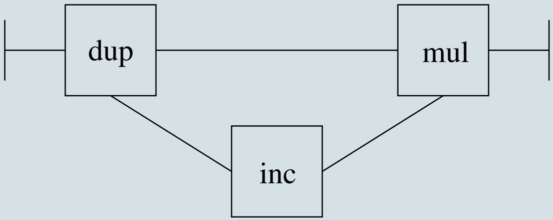
Beyond Petri Nets

Connected places:



$$P^2 = \nabla_{\{\,\underline{put_i},pass_k,\underline{get_j}\,\}}(\Gamma_{\{\,\underline{get_k}|\underline{get_k}\to pass_k\,\}}(P_{i,k}(\emptyset) \parallel P_{k,j}(\emptyset)))$$

Connected transitions:



mCRL2 data language

Is it advantageous to use an existing data language?
Not likely, because:

- algebraic specification languages are often *first-order* and lack *concrete data types*
- functional programming languages cannot handle *open terms* and are focused on *evaluation* only
- it is often hard to integrate an existing language in a toolset

mCRL2 data language (2)

Conclusion: we define our own language, but keep the door open to existing algebraic specification languages.

Approach:

- define a core theory of higher-order algebraic specification
- add concrete data types:
  - add syntax
  - implement data types within the core theory

Higher-order algebraic specification

Concepts: sorts, operations, terms and equations

Higher-order *sorts* are constructed as follows, where b is a set of *base* sorts:

$$s := b \mid s \rightarrow s$$

An *operation* is of the form $f : s$, which means that all operations are constants.

Data *terms* are constructed from variables and operations:

$$d ::= x : s \mid f : s \mid d(d)$$

Higher-order algebraic specification (2)

We use a *conditional equational logic* to express properties of data:

$$\phi ::= \forall \overrightarrow{x{:}s}.\ d = d\ \wedge \cdots \wedge\ d = d \rightarrow d = d$$

Data specification elements:

$$\begin{aligned} dse ::= &\ \textbf{sort}\ (b;\ )^+ \\ |&\ \textbf{cons}\ (f : s;\ )^+ \\ |&\ \textbf{map}\ (f : s;\ )^+ \\ |&\ (\textbf{var}\ (x : s;\ )^+)?\ \textbf{eqn}\ (\phi;\ )^+ \end{aligned}$$

Data specification:

$$ds ::= dse^*$$

HOAS in practise

Changes/additions:

- conditional equations are restricted to $d \to d = d$, where the condition is a term of predefined sort $\mathbb{B}$
- $s_0 \times \cdots \times s_n \to s$ is a shorthand for $s_0 \to \cdots \to s_n \to s$, where $\to$ is right-associative
- $t(t_0, \ldots, t_n)$ is a shorthand for $t(t_0) \cdots (t_n)$, where application is left-associative
- sort references can be defined:

$$\textbf{sort}\ B = C \to D;$$

- add prefix, infix and mixfix notation for concrete data types, together with operator precedence

HOAS in practise: concrete data types

General:

- equality $d == d$, inequality $d \neq d$ and conditional $\mathit{if}(d, d, d)$
- lambda expressions $\lambda \overrightarrow{x{:}s}.d$
- where clauses d **whr** $x = d, \ldots, x = d$ **end**

Basic data types:

- Booleans ($\mathbb{B}$)
  $\mathit{true}, \mathit{false}, \neg d, d \wedge d, d \vee d, d \Rightarrow d, \forall \overrightarrow{x{:}s}.d, \exists \overrightarrow{x{:}s}.d$
- Numbers ($\mathbb{P}$, $\mathbb{N}$ and $\mathbb{Z}$)
  $0, 1, -1, 2, -2, \ldots$
  $d < d, d \leq d, d > d, d \geq d, -d, d + d, d - d, d * d, d \,\mathbf{div}\, d, d \,\mathbf{mod}\, d, \ldots$

HOAS in practise: concrete data types (2)

Type constructors:

- structured types (sum types and product types)

$$
\begin{array}{rl}
\textbf{struct} & c_1(pr_{1,1} : A_{1,1},\ \ldots, pr_{1,k_1} : A_{1,k_1})?is\_c_1 \\
 & \mid c_2(pr_{2,1} : A_{2,1},\ \ldots, pr_{2,k_2} : A_{2,k_2})?is\_c_2 \\
 & \vdots \\
 & \mid c_n(pr_{n,1} : A_{n,1}, \ldots, pr_{n,k_n} : A_{n,k_n})?is\_c_n
\end{array}
$$

- lists ($List(s)$)
  $[\,], [d, \ldots, d], \#d, d \triangleright d, d \triangleleft d, d +\!\!+ d, d.d$
- sets and bags ($Set(s)$, $Bag(s)$)
  $\emptyset, \{\, d, \ldots, d \,\}, \{\, d{:}d, \ldots, d{:}d \,\}, \{\, x{:}s \mid d \,\}$
  $\#d, d \in d, d \subseteq d, d \subset d, d \cup d, d \setminus d, d \cap d, \overline{d}$

Example: Sliding Window Protocol

```
map  n: Pos;

sort D = struct d1 | d2;
     Buf = Nat -> struct data(getdata:D) | empty;
map  emptyBuf: Buf;
     insert: D#Nat#Buf -> Buf;
     remove: Nat#Buf -> Buf;
     release: Nat#Nat#Buf -> Buf;
     nextempty: Nat#Buf -> Nat;
     inWindow: Nat#Nat#Nat -> Bool;
var  i,j,k: Nat; d: D; q: Buf;
eqn  emptyBuf = lambda j:Nat.empty;
     insert(d,i,q) = lambda j:Nat.if(i==j,data(d),q(j));
     remove(i,q) = lambda j:Nat.if(i==j,empty,q(j));
     release(i,j,q) =
        if((i mod 2*n)==(j mod 2*n),
           q,
           release((i+1) mod 2*n,j,remove(i,q)));
     nextempty(i,q) = if(q(i)==empty,i,nextempty((i+1) mod n,q));
     inWindow(i,j,k) = (i<=j && j<k) || (k<i && i<=j) || (j<k && k<i);
```

Example: Sliding Window Protocol (2)

```
act   sA,rA,sD,rD: D;
      sB,rB,cB,sC,rC,cC: D#Nat;
      sE,rE,cE,sF,rF,cF: Nat;
      j;

proc S(l,m:Nat,q:Buf) =
         sum d:D. inWindow(l,m,(l+n) mod 2*n) ->
                  rA(d).S(l,(m+1) mod 2*n,insert(d,m,q))+
         sum k:Nat. (q(k)!=empty) -> sB(getdata(q(k)),k).S(l,m,q)+
         sum k:Nat. rF(k).S(k,m,release(l,k,q));

     R(l:Nat,q:Buf) =
         sum d:D,k:Nat. rC(d,k).
                  (inWindow(l,k,(l+n) mod 2*n) -> R(l,insert(d,k,q)),R(l,q))+
         (q(l)!=empty) -> sD(getdata(q(l))).R((l+1) mod 2*n,remove(l,q))+
         sE(nextempty(l,q)).R(l,q);

     K = sum d:D,k:Nat. rB(d,k).(j.sC(d,k)+j).K;

     L = sum k:Nat. rE(k).(j.sF(k)+j).L;

init allow({cB,cC,cE,cF,j,rA,sD},
         comm({rB|sB->cB, rC|sC->cC, rE|sE->cE, rF|sF->cF},
            S(0,0,emptyBuf) || K || L || R(0,emptyBuf)));
```

Implementation of concrete data types

General requirements:

- computability: reading the equations from left to right, we obtain a term rewrite system that is confluent, terminating and complete (if possible)
- simplicity: internal representation should be unique
- efficiency:
  - reduction lengths should be minimised
  - the number of equations should be minimised
- provability: the number of properties that can be proved on open terms should be maximised

Implementation of concrete data types (2)

Data type specific:

- lambda expressions and where clauses are implemented as named functions, e.g. $\lambda y{:}\mathbb{N}.(x+y)$ becomes $f(x)$, where $f : \mathbb{N} \to \mathbb{N} \to \mathbb{N}$ satisfies $f(x)(y) = x + y$, for all $x, y : \mathbb{N}$
- quantifications over sort s are implemented as functions of sort $(s \to \mathbb{B}) \to \mathbb{B}$
- numbers have a unique binary representation:
  - sort $\mathbb{P}$ has constructors $1 : \mathbb{P}$ and $cDub : \mathbb{B} \times \mathbb{P} \to \mathbb{P}$
  - sort $\mathbb{N}$ has constructors $0 : \mathbb{N}$ and $cNat : \mathbb{P} \to \mathbb{N}$
  - sort $\mathbb{Z}$ has constructors $cInt : \mathbb{N} \to \mathbb{Z}$ and $cNeg : \mathbb{P} \to \mathbb{Z}$
- sets and bags over sort s are implemented as functions $s \to \mathbb{B}$ and $s \to \mathbb{N}$

Linear process equations

μCRL LPE:

$$P(\overrightarrow{d{:}D}) = \sum_{i\in I} \sum_{\overrightarrow{e_i{:}E_i}} a_i(\overrightarrow{f_i}(\overrightarrow{d, e_i})) \cdot P(\overrightarrow{g_i}(\overrightarrow{d, e_i})) \lhd c_i(\overrightarrow{d, e_i}) \rhd \delta$$

mCRL2 LPE:

$$P(\overrightarrow{d{:}D}) = \sum_{i\in I} \sum_{\overrightarrow{e_i{:}E_i}} c_i(\overrightarrow{d, e_i}) \rightarrow$$
$$(a_i^0(\overrightarrow{f_{i,0}}(\overrightarrow{d, e_i})) \mid \cdots \mid a_i^{n(i)}(\overrightarrow{f_{i,n(i)}}(\overrightarrow{d, e_i}))) \cdot t_i(\overrightarrow{d, e_i}) \cdot P(\overrightarrow{g_i}(\overrightarrow{d, e_i})),$$

where:

- data types are higher-order
- *free* variables are used to model don't care values

Tool support

Because of the changes to the core language (LPEs), reuse of existing tools is hard. So we re-implemented some of them.

New goals:

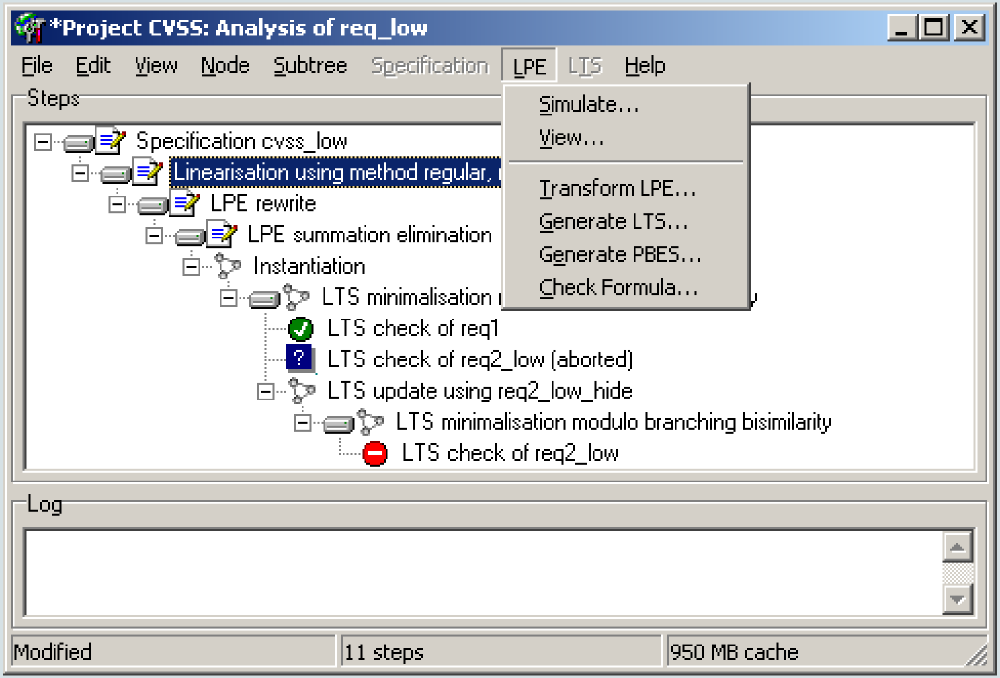
- graphical user interface that will:
  - lower the treshold for new users
  - simplify the analysis process
- flexible LPE simulator with different pluggable views
- model checking directly on LPEs
- visualisation of large LTSs

GUI: Analysis interface

Features:

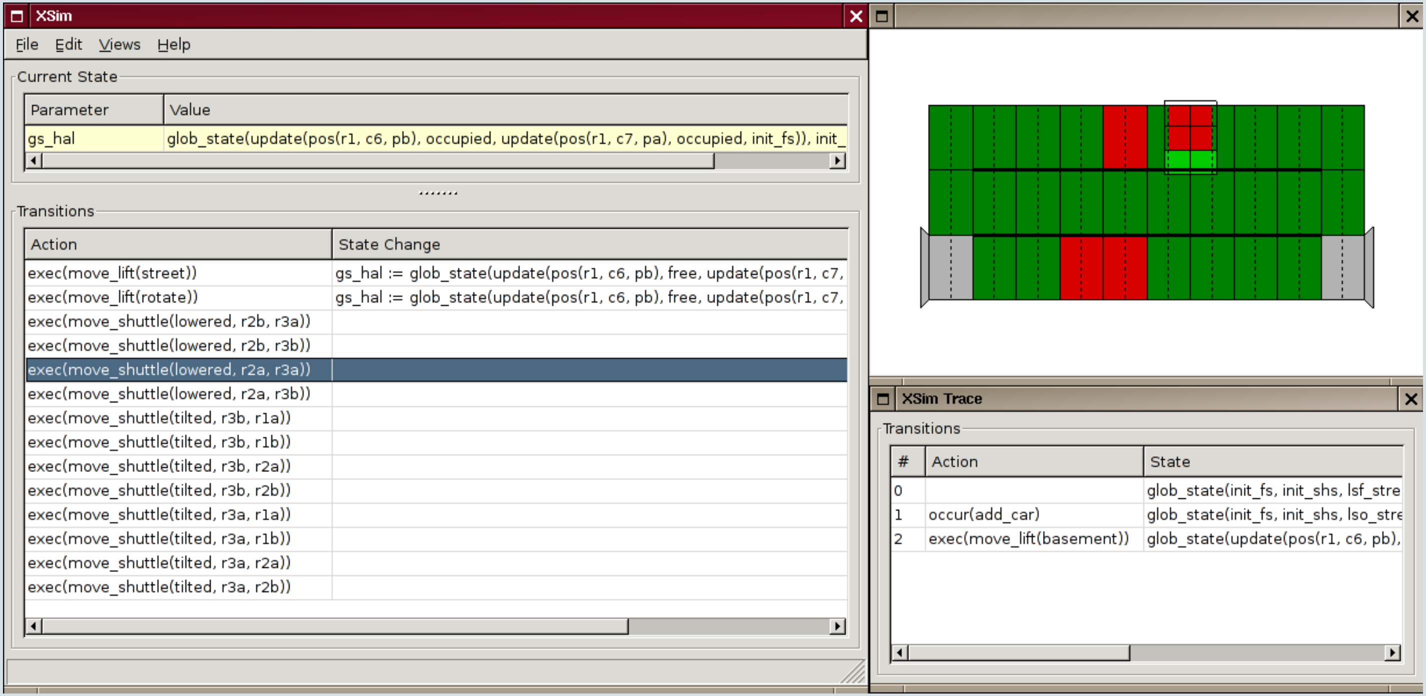
- tree represents an analysis:
  - each node is labelled with the result of an analysis step
  - each analysis step corresponds to the execution of a tool
- parameters can be supplied to tools using a graphical interface
- analysis trees abstract from temporary files: treated as cache

GUI: Analysis interface (2)



Graphical simulator

Features: simulate LPEs, pluggable views



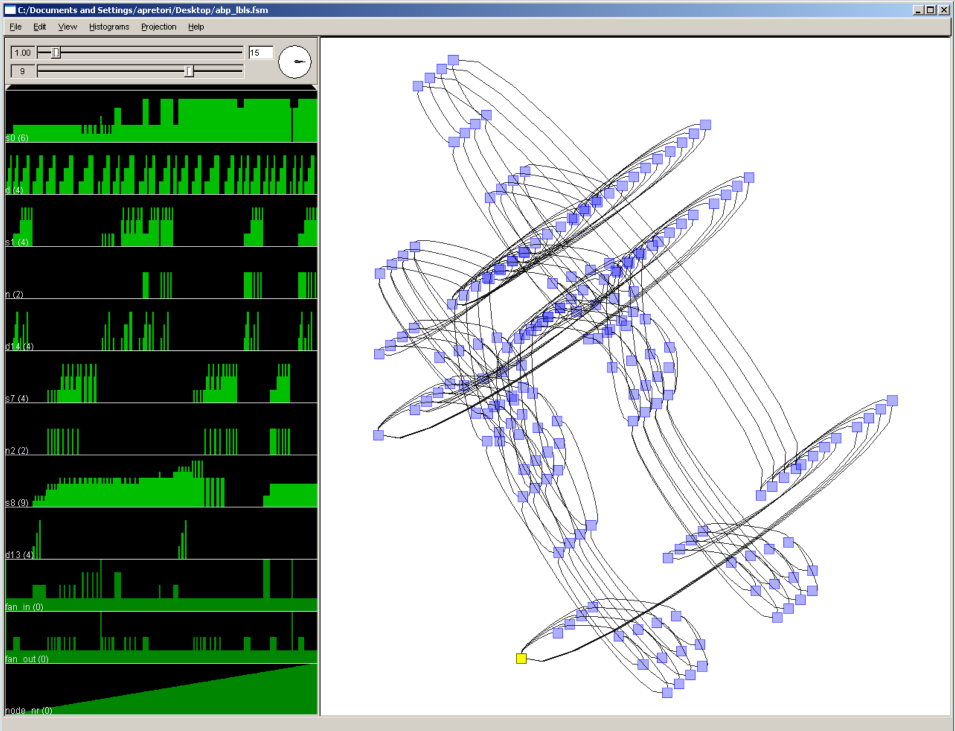
Model checking on LPEs

Parameterised Boolean Equation Systems (PBESs): mixture of BES and HOAS

Technique: LPE + property $\longrightarrow$ PBES $\longrightarrow$ BES

Muck van Weerdenburg, Jan Friso Groote

Visualisation of large LTSs (Hannes Pretorius)



Tool development status

Finished (mostly):

- parser
- type checker
- implementation of concrete data types
- lineariser
- rewriter
- simulator (both textual and graphical)
- instantiator
- 2D LTS visualiser

Tool development status (2)

To be implemented:

- LPE reduction tools
- LPE model checker
- graphical analysis interface
- prover
- Petri Net to mCRL2 convertor
- μCRL to mCRL2 convertor and vice versa

Conclusions and future work

mCRL2 is an attempt to make μCRL more applicable in practise.
It is extended such that:

- Petri Nets can be facilitated
- the treshold for new users is lowered

Future work:

- formalise the syntax and semantics of mCRL2
- finish the toolset and apply it to a number of real world cases
- find a connection between the mCRL2 toolset and other toolsets